

AP Calculus BC Summer Packet

Review of Derivatives, Integrals, and Applications. No Calculator. Show all Work.

Write out the derivative with respect to x of each of the following.

1. $y = \sin x$

2. $y = \cos x$

3. $y = \tan x$

4. $y = e^x$

5. $y = \csc x$

6. $y = \sec x$

7. $y = \cot x$

8. $y = \sin^{-1} x$

9. $y = \tan^{-1} x$

10. $y = \sec^{-1} x$

11. $y = \ln x$

12. $y = \log_a x$

13. $y = a^x$

Find the derivative of the following.

14. $y = (x + 2)^8(x + 3)^6$

15. $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

16. $y = \frac{x}{\sqrt{9-4x}}$

17. $y = \frac{e^x}{1+x^2}$

18. $y = \sin(\cos x)$

19. $y = \sin^{-1} e^x$

20. $y = xe^{\frac{1}{x}}$

21. $y = x^r e^{sx}$ (r and s constants)

22. $y = \tan \sqrt{1-x}$

23. $y = \frac{1}{\sin(x-\sin x)}$

24. $y = \ln(\csc 5x)$

25. $y = \ln(x^2 e^x)$

26. $y = 5^{x \cdot \tan x}$

27. $x^2 y^3 + 3y^2 = x - 4y$

28. $y = \arctan(\arcsin \sqrt{x})$

29. $f(x) = [g(x)]^2$

30. $\int_1^{x^2} \sqrt{1+t^3} dt$

Find the definite or indefinite integral as indicated.

31. $\int_0^1 (1-x^9) dx$

32. $\int (1-x)^9 dx$

33. $\int \frac{x}{x^2+1} dx$

34. $\int \frac{1}{x^2+1} dx$

35. $\int \frac{x^2-x+1}{\sqrt[3]{x^2}} dx$

36. $\int x^3 \ln x dx$

37. $\int \frac{1}{2-3x} dx$

38. $\int \sin x \cdot \cos(\cos x) dx$

39. $\int x^2 e^{-3x} dx$

40. $\int e^x \cos x dx$

41. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

42. $\int \frac{x}{9+x^4} dx$

43. $\int \frac{1-3y}{\sqrt{2y-3y^2}} dy$

44. $\int \cos 3x \, dx$

45. $\int t \cos(2t)^2 \, dt$

46. $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

47. $\int \sec \frac{t}{2} \, dt$

48. $\int \frac{\ln v}{v} dv$

49. $\int \ln x \, dx$

50. $\int \frac{y-1}{y+1} dy$

51. $\int \frac{5x-4}{2x^2+x-1} dx$

Applications

52. The angle of elevation of the sun is decreasing at a rate of 0.25 rad/hr. How fast is the shadow cast by a 400 ft building increasing when the angle of elevation is $\frac{\pi}{6}$?

53. A liquid form of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of \$200 per unit. If the total production cost (in dollars) for x units is $C(x) = 500,000 + 80x + 0.003x^2$, and knowing the Profit = Revenue – Cost, how many units of penicillin must be manufactured and sold to maximize profit?

54. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cubic cm/s, how fast is the water level rising when the water is 5 cm deep?

55. The rate at which sewage enters a treatment tank is given by $E(t) = 850 + 715\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \leq t \leq 4$ hours. How many gallons of sewage are in the tank after 4 hours if 250 gallons are in there at time $t = 0$?

56. The rate of change of bacteria growth is proportional to the amount present. If the count was 400 after 2 hours and 25,600 after 6 hours ($y =$ amount present, $\frac{dy}{dx}$ is the rate of growth, $\frac{dy}{dx} = ky$ is the rate of growth proportional to the amount present where k is the constant of proportionality),

a. What was the initial population of the culture?

b. Find an expression for the population after t hours.

c. In what period of time does the population double?

d. When will the population reach 100,000?

Find the solution to the differential equations.

57. $\frac{dy}{dx} = y^2 + 1$

58. $\frac{dy}{dx} = \frac{1+x}{xy}$ $y(1) = -4$

59. $x + 2y\sqrt{x^2 + 1}\frac{dy}{dx} = 0$ $y(0) = 1$